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Should we transfer resources from college to basic education?

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Abstract We analyze public intervention in two educational levels: basic education and college education. The government decides per capita expenditure at each level and the subsidy for college education. We explore the effect of transferring money from one level to the other on equity and efficiency. We prove the existence of an Equity-Efficiency Frontier (EEF), and analyze which policy reforms are optimal when the society is not at the EEF. For developed countries, this policy consists of transferring resources from college education to basic education.

Keywords Basic education · College education · Public expenditure in education

JEL Classification H52 · I28 · J24

1 Introduction

In most countries, public expenditure on education accounts for a large proportion of total expenditure on education. For the OECD countries, an average of 87% of expenditure on all levels of education came from public sources in 2004.[1] Public intervention is present at all educational levels, from pre-primary to tertiary education. However, countries differ dramatically according to how they allocate resources

[1] See Table B3.1, Education at a Glance 2007, OECD.

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Table 1 Public expenditure on education, 2004

Country	Expenditure per student[a]			Change 1995–2004[b]			Subs.[c]	C. Att.[d]	Exp.[e]
	Basic	Tertiary	T/B	Basic	Tertiary	T/B			
Austria	106,396	73,983	0.70		122		93.7	37	120.063
Belgium	86,320	35,406	0.41				90.4	34	99.317
Denmark	109,777	56,332	0.51	121	123	1.02	96.7	55	131.468
Finland	79,900	60,659	0.76	122	110	0.90	96.3	73	101.830
France[f]	86,406	42,884	0.50				83.9	39	100.551
Germany	87,659	65,732	0.75	105	107	1.02	86.4	37	100.437
Greece	58,850	29,361	0.50	192	151	0.79	97.9	33	66.157
Hungary	47,469	36,353	0.77	157	73	0.47	79.0	68	53.090
Iceland	113,213	32,770	0.29				90.9	79	123.877
Ireland	82,479	33,083	0.40	181	126	0.70	82.6	44	93.587
Italy	103,871	55,751	0.54	105	130	1.23	69.4	55	110.102
Japan	84,930	49,624	0.58	127	101	0.79	41.2	43	95.804
Korea	67,567	24,242	0.36				21.0	48	70.162
Mexico	22,662	19,761	0.87	130	110	0.85	68.9	29	25.128
Netherlands	74,339	72,555	0.98	136	101	0.75	77.6	56	94.256
New Zealand	74,745	27,042	0.36				60.8	89	79.803
Slovak Republic	32,856	25,484	0.78	155	111	0.71	81.3	47	36.230
Spain	69,993	43,699	0.62	136	167	1.24	75.9	44	83.171
Sweden	92,979	75,901	0.82	117	99	0.84	88.4	79	117.997
Turkey	15,396	12,474	0.81				90.0	26	16.724
UK	81,732	49,872	0.61	120	93	0.78	69.6	52	93.896
OECD average	75,216	43,951	0.61	138	109	0.79	78.2	32	86.364

[a] Cumulative expenditure per student in 2004. In equivalent US dollars converted using PPPs for GDP. Source: Education at a Glance 2007, Table B1.3b and authors' calculations

[b] Index of change in annual expenditure per student in US dollars, using PPPs, setting expenditure in 1995 at 100. Source: Education at a Glance 2007, Tables B1.1a and B1.5 and authors' calculations

[c] Proportion of public expenditure in tertiary education in 2004. Source: Education at a Glance 2007, Table B3.2b

[d] Entry rates into tertiary-type A programmes for 2004. Source: Education at a Glance 2006, Table C2.1

[e] Total expenditure on education. Authors' calculations

[f] Year of reference for C. Att.: 2003

across the different educational levels. Columns 1 and 2 of Table 1 show data compiled by the OECD on *cumulative* expenditure per student at basic and tertiary education, respectively.[2] In Column 3 we compute the ratio between expenditure in tertiary education and in basic education. We observe a large heterogeneity. The ratio ranges from 0.36 in Korea and New Zealand to 0.98 in The Netherlands, with an average of 0.61. Columns 4 and 5 show the change in annual expenditure per student from 1995 to

[2] Cumulative expenditure takes into account the duration of each educational level. Basic education corresponds to primary, secondary, and post-secondary non-tertiary education.

2004 and the ratio between both indexes is reported in Column 6. Ten countries out of fourteen have a ratio lower than one, meaning that in this period they have diverted resources from tertiary to basic education, at least in relative terms.

Our paper studies how governments should divide resources between basic (compulsory) education and higher (non-compulsory) education. To do this, it is crucial to discuss which objectives a government may have in its educational policy. Most governments care for efficiency and equity issues in a wide sense. However, sometimes the problem is to give a precise meaning to these general principles. To circumvent this problem we propose that equity concerns imply that the objective of the government should be to facilitate for everybody the access to education, irrespective of family background. Regarding efficiency, first we see that Pareto efficiency has no bite in our model (see Sect. 3.2). That is, except for very extreme cases, most feasible policy combinations pass the Pareto test. Then, we propose two alternative efficiency objectives: to maximize college students productivity and to maximize average productivity of the whole population.[3] One of the contributions of our paper is that we explicitly define an equity objective. This contrasts with most of the literature that either cares only for efficiency, avoiding any statement about equity, or weighs equity and efficiency concerns in the way prescribed by an (ad-hoc) social welfare function.[4] In addition, we study how these two objectives relate to each other and we analyze which policies the government should implement to achieve efficiency and equity at the same time. In particular, we want to study whether both objectives are compatible or not and, if they are, which policies make them compatible. Second, we explore whether all countries, rich and poor, should apply the same policy to satisfy these two objectives or if the policy reform is country-specific.

Our model has two educational stages: basic and college education. Basic education comprises all mandatory levels of education and it is fully financed by the government. In contrast, college education is voluntary and students may have to pay a part of the cost. Another difference is that expenditure on basic education affects the quality of education, but not enrollment, since attendance is mandatory. On the other hand, expenditure on college education affects not only quality, but also enrollment. Individuals who go to college get a skilled job, while the rest remain unskilled. Due to capital markets imperfections, some individuals suffer from borrowing constraints.

In our model, any public policy is fully characterized by two variables corresponding to expenditure on basic and college education, respectively. We define the Equity-Efficiency Frontier (EEF) as the set of public policies for which it is not possible to improve the two objectives of the government at the same time. The idea is similar to that of the Pareto set in an Edgeworth Box. In general, except by chance, we should not expect the economy to be at the EEF. Then, it is interesting to study if we can find a policy reform that simultaneously satisfies the objectives of increasing equity and efficiency. We prove that this is always the case. We also find that for rich countries, this policy consists of transferring resources from college to basic education. The intuition

[3] See for example Lloyd-Ellis (2000), Su (2004) and Blankenau et al. (2007) who consider similar criteria.

[4] There are, however, notable exceptions such as Keane and Roemer (2009), who evaluate education policies aimed at equalizing opportunities.

is that this policy reduces the threshold level of income needed to attend college, but at the price of raising the threshold level of ability. Since higher education is heavily subsidized in the rich countries, the first effect is smaller in size and attendance falls. However, due to the increase in the threshold level of ability, the productivity of skilled workers rises. In addition, this policy has a positive effect on the productivity and the number of unskilled workers. Therefore, we also find that by transferring resources from college to basic education the average productivity across the population as a whole rises. On the contrary, for low income countries the policy reform that has a positive effect on equity and at the same time improves the productivity of skilled workers consists of transferring resources from basic to college education. However, in general this policy will have a negative effect on the average productivity across the population.

Note that we focus on educational reforms, instead of focusing on the design of an optimal educational policy. We start from a given division of the budget between the two levels of education and we study the effect of diverting resources from one educational level to the other. We believe that this is a sensible approach since most governments, instead of introducing large reforms, introduce small reforms in several steps. In addition, it is commonly accepted in much of the literature on optimal commodity taxation which focuses on local effects of tax changes (see Feldstein 1975; Guesnerie 1977; King 1983; Mayeres and Proost 2001 and references therein).

We discuss briefly some previous works related to ours. Lloyd-Ellis (2000) studies the impact of alternative allocations of public resources between basic and higher education on enrollment, income distribution and growth, while Blankenau et al. (2007) investigate its output and welfare implications. However, none of them consider individual heterogeneity with respect to parental income, which is one of our main focuses. Driskill and Horowitz (2002) study optimal investment in human capital in a standard growth model, and they find that developing countries should concentrate on advanced human capital, a result similar to ours. Restuccia and Urrutia (2004) focus on intergenerational mobility and find that an increase in expenditure on early education has more impact than an increase in college subsidies. Su (2004) studies the dynamic effects of allocating public funds between basic and college education. However, she abstracts from private education expenditure which is a crucial factor affecting education outcomes. Gilboa and Justman (2009) study the different effects of university tuition and student loan policies on university graduation and enrolment rates and on total output. However, they confine their analysis to higher education expenditures without considering the impact that such policies have on public expenditure on basic education.

Finally, Romero (2008) considers that voters decide how to split the budget between basic and college education and he studies how the possibility of opting out from public education affects that decision.

The paper is organized as follows. In Sect. 2 we describe the economy. In Sect. 3 we consider the effect of public policies on the different objectives of the government and we illustrate our main result with a numerical example. In Sect. 4 we discuss the robustness of the main results to alternative assumptions. Finally, Sect. 5 concludes.

2 Model

2.1 Individuals and educational sector

We build a model with two periods and a continuum of individuals characterized by parental income $y \in [0, Y]$ and innate ability $a \in [0, A]$, where $Y, A > 0$. The cumulative distribution functions are $F(y)$ and $G(a)$ respectively, although to get closed-form solutions we will assume that a is uniformly distributed on its support.[5] We also assume that y and a are independently distributed. As we will see below, college attendance is the proportion of individuals with ability and income above some given thresholds. The assumption that a and y are independently distributed allows us to study separately the effect of policy changes on college attendance through the effect on the two thresholds.

In the first part of the first period all children attend compulsory basic education. In the rest of the first period, individuals can either get a job as unskilled workers or enrol in higher education to become skilled workers. We assume that it takes a fraction of time δ of the first period to get a college degree. This parameter δ also represents the amount of time low-skilled agents work in the first period.

In the second period, individuals with a college degree get a skilled job, while all others remain in an unskilled job. Individuals care for their consumption in the second period (C) which is equal to the value of their lifetime income.

We assume a simple structure for the educational sector. The per capita cost of providing basic education is $c_L > 0$. Since basic education is compulsory, we assume that its cost is paid in full by the government.[6] Regarding higher education, the level of public provision is $c_H > 0$, which is the per capita cost of providing higher education. This includes wages paid to teachers, the cost of college equipment, laboratories, etc. The proportion of the total cost that the government pays is s, with $0 \leq s \leq 1$. That is, the government pays $c_H s$, while students pay $c_H(1 - s)$. To simplify things, we assume that the subsidy is the same for all individuals. In Sect. 4 we discuss the possibility of having subsidies depending on family income. Finally observe that, as in Blankenau et al. (2007) and Lloyd-Ellis (2000), we do not consider the existence of fixed costs. One reason is of tractability. Another reason is that we are interested only in marginal changes in per capita costs.

We want to distinguish between public provision and public financing as each one of them can be used by the government to achieve different objectives. The parameter c_H, as well as c_L in the case of basic education, captures the quality of education. Increasing c_H could be seen as a way of improving the quality of college education which, in turn, may have a positive effect on the human capital of college graduates. However, for a fixed level of c_H, an increase in s can be seen as a way of easing access to college for individuals from low-income families.

[5] This assumption simplifies a lot the exposition. In the numerical examples, results are not very different using a Lognormal distribution.

[6] In 2003, only 7.4% of total expenditure in basic education in the OECD (primary, secondary and post-secondary non-tertiary education) was privately financed.

2.2 College attendance

Since individuals care only for their consumption in the second period C, they will maximize lifetime income. An individual who only attends basic education will be an unskilled worker forever. We assume that her productivity and, thus, her wage w_U will be determined exclusively by per capita expenditure at basic education c_L. We write $w_U = w_U(c_L)$, and we assume this function to be increasing and weakly concave. Since they work a fraction δ of the first period, their lifetime income is $(1+\delta)w_U(c_L)$. To simplify further the analysis we assume that individuals do not discount future payoffs.

The wage paid to a college graduate is $w_S(c_L, c_H, a)$, which is increasing and weakly concave with respect to the three arguments. In other words, investment in education has a positive effect on productivity. This is a standard assumption in the literature (see Restuccia and Urrutia 2004; Blankenau et al. 2007 among others). In Sect. 4 we discuss the assumption that ability does not affect unskilled wages. Lifetime income of a skilled worker will be $w_S(c_L, c_H, a) - c_H(1-s)$. An individual will choose higher education if:

$$w_S(c_L, c_H, a) \geq c_H(1-s) + (1+\delta)w_U(c_L). \tag{1}$$

An interior equilibrium will be characterized by a threshold level of ability $\widehat{a}$ such that:

$$w_S(c_L, c_H, \widehat{a}) = c_H(1-s) + (1+\delta)w_U(c_L). \tag{2}$$

Individuals who want to attend college must have enough resources to pay the tuition cost $c_H(1-s)$. They can use their income y and they have also access to a loan from a bank. Due to capital market imperfections, we assume they can borrow only up to an amount $\gamma c_H(1-s)$, where $0 \leq \gamma \leq 1$.[7] The parameter γ captures the "quality" of capital markets. The higher is γ the better is the quality of capital markets. However, γ could alternatively be interpreted as a policy variable. Many countries are offering students' loans to overcome this constraint. Then $\gamma = 1$ means that there is such a policy in place, while $\gamma = 0$ means a complete absence of it. This borrowing constraint, an exogenous feature of the model, is assumed to be the same across individuals.[8]

[7] Evidence by Cameron and Taber (2004) and others regarding the United States suggests that credit constraint are not important in determining college attendance. However, access to the most prestigious (and expensive) programs remains conditioned on socioeconomic background. In Sect. 4 we discuss how the main results of the paper change with perfect capital markets.

[8] Gilboa and Justman (2009) assume that students from poor families face a higher rate of interest than students from more affluent families. Lochner and Monge-Naranjo (2010) build a model with endogenous borrowing constraints. Individuals of heterogeneous abilities or those making different schooling choices face different borrowing constraints. We implicitly assume that banks cannot condition loans on ability, as they cannot observe it.

To attend college, therefore, individuals must have pre-tax income satisfying:

$$y \geq \widehat{y}(c_H, s) = (1-\gamma)c_H(1-s). \tag{3}$$

Those with income above $c_H(1-s)$ do not need to ask for a loan. Those with income below $(1-\gamma)c_H(1-s)$ cannot afford college. Those with income between $(1-\gamma)c_H(1-s)$ and $c_H(1-s)$ need a loan to attend college. The proportion of individuals who can afford college is the proportion of individuals with pre-tax income above $\widehat{y}$, namely, $1-F(\widehat{y})$. When $\gamma = 1$ or $s = 1$, we get $\widehat{y} = 0$ and the constraint is not binding for any individual. To simplify notation we call $p = 1-F(\widehat{y})$, and in what follows we assume that $p > 0$. The proportion of college students π is the proportion of individuals who satisfy conditions (1) and (3). Since y and a are independently distributed, we have:

$$\pi(c_L, c_H, s) = \frac{(A-\widehat{a})}{A} \times p. \tag{4}$$

Using (2) and (3) we can rewrite this as:

$$\pi(c_L, c_H, s) = \frac{(A-\widehat{a}(c_L, c_H, s)}{A} \times [1 - F[(1-\gamma)c_H(1-s)]]. \tag{5}$$

It is immediate to check that π is increasing with γ and s. The effect of c_L on π will be negative if the impact of c_L on $\widehat{a}$ is positive. Finally, to analyze the effect of c_H on π we see that an increase in c_H raises the tuition cost reducing p, while the effect on the term $(A-\widehat{a})$ depends on whether the impact of c_H on $\widehat{a}$ is negative or not. If it is negative, these two effects go in opposite directions and the final effect will depend on which of the two effects prevail. If it is positive, the effect of c_H on π will be unambiguously negative. However, we want to stress that these results are just partial derivatives, since we are not taking into account the budget constraint of the government. In the Appendix we discuss the total effect of c_L and c_H on $\widehat{a}$.

2.3 The government budget constraint

Here we study how the three instruments of the government (c_L, c_H, s) are related through the budget constraint. Total expenditure per capita in education is $E \equiv c_L + sc_H\pi$. We call T the total educational budget per capita which we assume to be fixed. Since we assume that the government cannot run a deficit, the constraint is:

$$E \equiv c_L + sc_H\pi \leq T. \tag{6}$$

For fixed values of c_L and c_H, we call $\widehat{s}(c_L, c_H)$ the value of the subsidy for which the constraint is satisfied with equality:

$$E \equiv c_L + \widehat{s}(c_L, c_H)c_H\pi(c_L, c_H, \widehat{s}(c_L, c_H)) = T. \tag{7}$$

If $E < T$ for all values of the subsidy, then we set $\widehat{s}(c_L, c_H) = 1$. In the lemma below we provide a simple condition that guarantees existence and uniqueness of $\widehat{s}(c_L, c_H)$.

Lemma 1 *Consider any combination* (c_L, c_H) *and assume that* $T > c_L$. *Then, there is a unique value* $\widehat{s}(c_L, c_H) \leq 1$ *that satisfies the budget constraint. If* $E < T$ *for all* s, *then* $\widehat{s}(c_L, c_H) = 1$.

Proof When $s = 0$, we have $E = c_L$. Since π is increasing in s, the function E is strictly increasing in s. We have two possibilities. Either E is always below T, in which case $\widehat{s}(c_L, c_H) = 1$, or they cross at a value of the subsidy $\widehat{s}(c_L, c_H)$ strictly below 1. □

The condition $T > c_L$ is required since, otherwise, even when college education is not subsidized at all, the government would be running a deficit. Since the government has to satisfy the budget constraint, it has only two free policy instruments. We choose c_L and c_H as the two free parameters and we assume that the subsidy always adjusts to satisfy the constraint. Since we are interested in policy changes, we want to study the effect of changes in c_L and c_H on $\widehat{s}(c_L, c_H)$. We focus on an interior equilibrium. Computing the corresponding derivatives:

$$\begin{aligned} \frac{d\widehat{s}}{dc_L} &= -\frac{\frac{\partial E}{\partial c_L}}{\frac{\partial E}{\partial s}} = -\frac{1 + \widehat{s} c_H \frac{\partial \pi}{\partial c_L}}{\frac{\partial E}{\partial s}}, \\ \frac{d\widehat{s}}{dc_H} &= -\frac{\frac{\partial E}{\partial c_H}}{\frac{\partial E}{\partial s}} = -\frac{\widehat{s}\left(\pi + c_H \frac{\partial \pi}{\partial c_H}\right)}{\frac{\partial E}{\partial s}}. \end{aligned} \tag{8}$$

Since $\frac{\partial E}{\partial s} \geq 0$, the signs of $\frac{d\widehat{s}}{dc_L}$ and $\frac{d\widehat{s}}{dc_H}$ will be negative if the terms in the numerator are positive. Consider first that college attendance π is not affected by either c_L or c_H. Then, both derivatives are negative. That is, raising either c_L or c_H reduces the resources that can be used to subsidize higher education. However, college attendance can also be affected negatively by the increase in c_L or c_H, reducing the absolute value in the numerator. Intuitively, the negative effect on the subsidy is attenuated, since now fewer individuals are subsidized. What we do is to assume that the indirect effect through π is not that large so as to offset the initial negative effect.

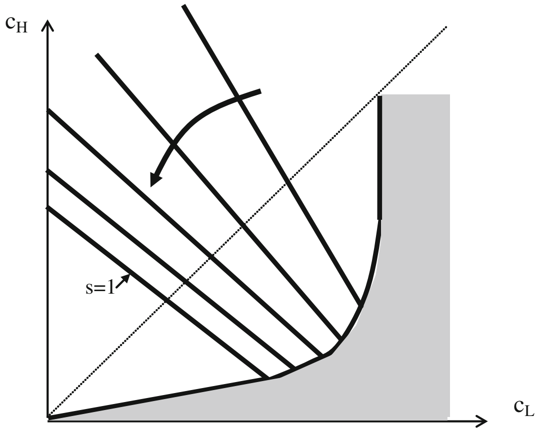
Assumption 1 (A.1): The following conditions hold: (i) $\frac{d\widehat{s}}{dc_H} < 0$ and (ii) $\frac{d\widehat{s}}{dc_L} < 0$.

Assumption 1 imposes that the government faces a trade-off when deciding between either c_L and s or between c_H and s. In the Appendix we discuss conditions that imply Assumption 1. Basically, we need to assume that the elasticities of π with respect to both c_L and c_H are of small size.

We define an *"iso-subsidy"* curve as the set of all combinations (c_L, c_H) giving rise to the same value of the subsidy $\widehat{s}$. From Eqs. (7) and (8), the slope of an iso-subsidy is:

$$\left.\frac{dc_H}{dc_L}\right|_{s=\widehat{s}} = -\frac{\frac{d\widehat{s}}{dc_L}}{\frac{d\widehat{s}}{dc_H}} = -\frac{1 + \widehat{s} c_H \frac{\partial \pi}{\partial c_L}}{\widehat{s}\left[\pi + c_H \frac{\partial \pi}{\partial c_H}\right]}. \tag{9}$$

Fig. 1 The policy space



By Assumption 1, this slope is negative, implying that there is always a trade-off between expenditure on basic education and expenditure on college education. Holding the subsidy fixed, if we increase one of them we have to reduce the other in order to keep the budget balanced.

In Fig. 1 we represent the iso-subsidy curves for a fixed value of the education budget T. The closer to the origin, the higher is the value of the subsidy. The area in grey represents combinations (c_L, c_H) where Eq. (1) does not hold even for the individual with highest ability A and, thus, nobody wants to attend college. This happens because c_H is extremely low and c_L is extremely high. From Lemma 1 we see that, if the education budget T rises, the subsidy corresponding to a given combination (c_L, c_H) will be higher. Moreover, college attendance will be higher as well.[9] We also find that, for a given combination (c_L, c_H), the iso-subsidy curve $\widehat{s}(c_L, c_H)$ is flatter the higher is T. The intuition is simple. Consider two countries A and B. In country A the budget is T and in country B it is $T' > T$. All other parameters of the model are assumed to take the same values in both countries. Then, any fixed combination $(\widehat{c}_L, \widehat{c}_H)$ will correspond to a higher value of the subsidy in country B than in country A. That is, $\widehat{s}'(\widehat{c}_L, \widehat{c}_H) > \widehat{s}(\widehat{c}_L, \widehat{c}_H)$. Moreover, college attendance must be also higher in country B than in country A. Now suppose we want to reduce c_H from $\widehat{c}_H$ to c'_H, holding the subsidy constant. How much we can raise c_L will depend on college attendance. In country A few people attend college, meaning that the reduction of c_H will save little money. We will be able to increase c_L only until c'_L. In country B college attendance is high. The same reduction of c_H will save much more money than in country A and we will be able to raise c_L until $c''_L > c'_L$.

[9] Note that once we choose a particular pair of values $(\widehat{c}_L, \widehat{c}_H)$, total expenditure in education $E = \widehat{c}_L + s\widehat{c}_H\pi(\widehat{c}_L, \widehat{c}_H, s)$ is strictly increasing in s (see the proof of Lemma 1). Then, from Eq. (7) in the paper we find that, the higher is T, the higher is the subsidy for which the constraint holds with equality. Finally, from Eqs. (5) and (7) we see that college attendance is also increasing with T.

3 Policy reforms

We analyze policy reforms from an initial situation described by a combination (c_L, c_H) through their effects on different government objectives. As we discussed in the introduction, we consider that the government wants to fulfill several objectives at the same time. In particular, it has both efficiency and equity concerns. Although we will be more precise below, by efficiency we refer to policies that improve the productivity of workers. By equity we mean policies that foster equality of opportunity.

3.1 Equity

For a fixed value of innate ability a, college attendance is determined only by income y. To facilitate college attendance for a given ability level a, the government has to reduce as much as possible the threshold level of income $\widehat{y}$ which, by our assumptions, is constant across ability levels. Then, to reduce the threshold $\widehat{y}$ amounts simply to reduce $c_H(1-\widehat{s})$. Since both $\frac{d\widehat{s}}{dc_L}$ and $\frac{d\widehat{s}}{dc_H}$ are negative, this can be done by either reducing c_L or c_H or both. In the space (c_L, c_H) we define an "iso-equity" curve as the set of combinations (c_L, c_H) giving rise to a constant level of $\widehat{y}$. Note that along any iso-equity curve the value of the subsidy changes so as to satisfy the budget constraint of the government. The slope of $\widehat{y}$ in the space (c_L, c_H) is:

$$\left.\frac{dc_H}{dc_L}\right|_{\widehat{y}=\overline{\widehat{y}}} = \frac{\frac{d\widehat{s}}{dc_L}}{\frac{(1-\widehat{s})}{c_H} - \frac{d\widehat{s}}{dc_H}}, \tag{10}$$

which is negative. This means that, if we increase c_H (respectively, c_L), to hold $\widehat{y}$ constant we have to reduce c_L (respectively, c_H). A government that only cares for equity should choose the lowest iso-equity level curve. However, once the government cares also for efficiency, in general that is not the best choice.

3.2 Efficiency

We focus first on Pareto efficiency. We find that in general almost all policy combinations (c_L, c_H) are Pareto efficient. That is, it is impossible to design feasible policy reforms that produce a Pareto improvement. This is why we need to propose alternative efficiency criteria. We begin by presenting a lemma where we present a necessary and sufficient condition for the existence of a Pareto improving reform. Since this condition in general will not hold, this can be interpreted as an impossibility result. Pareto efficiency has, therefore, no bite in our model.

Lemma 2 *Consider any given combination $(\widehat{c}_L, \widehat{c}_H)$. Then, there is a policy reform that leads to Pareto improvement if and only if the following condition holds:*

$$\varepsilon_{C_H}^{w_S} < \varepsilon_{C_L}^{w_S}\left(\frac{c_H}{c_L}\frac{\frac{d\widehat{s}}{dc_H}}{\frac{d\widehat{s}}{dc_L}}\right) + (1-\widehat{s})\frac{c_H}{w_S}, \tag{11}$$

where $\varepsilon^{w_S}_{C_H} = \frac{\partial w_S}{\partial c_H}\frac{c_H}{w_S}$ *and* $\varepsilon^{w_S}_{C_L} = \frac{\partial w_S}{\partial c_L}\frac{c_L}{w_S}$ *are elasticities of the skilled wage. Moreover, this policy consists of increasing* c_L *and reducing* c_H.

Proof Consider three curves that go through $(\widehat{c}_L, \widehat{c}_H)$: (i) a vertical line that represents all combinations for which lifetime income of unskilled individuals is constant; (ii) the curve that represents combinations for which the lifetime income of skilled individuals is constant and (iii) the iso-subsidy curve. Now consider the effects of local changes in c_L and c_H. First, any change to another combination (c_L, c_H), such that $c_L < \widehat{c}_L$ reduces lifetime utility for all unskilled workers. Second, consider any change to another combination (c_L, c_H), such that $c_L > \widehat{c}_L$. Since the slope of the iso-subsidy is negative, all combinations above the iso-subsidy that goes through $(\widehat{c}_L, \widehat{c}_H)$ are discarded, because they correspond to a higher value of the income threshold $\widehat{y}$, which implies that some individuals will be excluded from college attendance because they cannot afford it. These individuals will be worse off after the change, since they were high ability individuals who optimally decided to attend college before the policy change. It remains to study changes to combinations (c_L, c_H), such that $c_L > \widehat{c}_L$ that are below the iso-subsidy through $(\widehat{c}_L, \widehat{c}_H)$. With a change in this direction all the unskilled are better-off. There are also some individuals that now can afford college and, thus, are better-off as well. We only have to check what happens to those that go to college before and after the policy change. In fact, we have to study how lifetime income of skilled individuals changes with this reform. It will increase provided that the following condition holds:

$$-\frac{\left(\frac{\partial w_S}{\partial c_L} + c_H\frac{d\widehat{s}}{dc_L}\right)}{\left(\frac{\partial w_S}{\partial c_H} - (1-\widehat{s}) + c_H\frac{d\widehat{s}}{dc_H}\right)} < -\frac{\frac{d\widehat{s}}{dc_L}}{\frac{d\widehat{s}}{dc_H}}. \tag{12}$$

This condition says that the indifference curve of skilled lifetime income through $(\widehat{c}_L, \widehat{c}_H)$ is steeper than the iso-subsidy through that point. This condition can be easily simplified into Condition (11). □

Condition (11) tells us when should we expect an increase in the lifetime income of skilled workers after a reduction of c_H that goes together with an increase in c_L. This will happen only when $\varepsilon^{w_S}_{C_H}$ is much lower than $\varepsilon^{w_S}_{C_L}$. This does not seem to be the case for most reasonable technologies. To see this, note first that the last term on the right will be in general of small size, so we can disregard it. Then, we have to study the term that multiplies $\varepsilon^{w_S}_{C_L}$. The term $\frac{c_H}{c_L}$ is below 1 for most countries (see Table 1), while the term $\frac{d\widehat{s}}{dc_H}/\frac{d\widehat{s}}{dc_L}$ is the inverse (in absolute terms) of the slope of the iso-subsidy. In the Appendix we see that this is approximately $\widehat{s}\pi$ which is also lower than 1. Then, as long as $\varepsilon^{w_S}_{C_H}$ is larger than $\varepsilon^{w_S}_{C_L}$, Condition (11) will not hold. This implies that, in most cases, this policy change will reduce lifetime income of skilled workers. There is no possibility of a Pareto improvement.

Since Pareto efficiency does not restrict the set of policies, we propose two alternative efficiency objectives. First, we consider that the government wants to increase the average productivity or the average human capital of college graduates. This would be the case if government is particularly concerned with improving the productivity of skilled workers. Since an individual with ability a who attends college has productivity $w_S(c_L, c_H, a)$, the average productivity of graduates, denoted by Q_S is:

$$Q_S(c_L, c_H) = E[w_S(c_L, c_H, a) \mid a > \widehat{a}(c_L, c_H)]. \tag{13}$$

An increase in either c_L or c_H has a positive effect on Q_S, which is assumed to be concave with respect to both c_L and c_H. There is a positive direct effect through w_S and also an indirect positive effect if the threshold $\widehat{a}$ rises as well. In the space (c_L, c_H) we can define an *"iso-productivity"* curve as the set of combinations (c_L, c_H) giving rise to the same level of Q_S. Similarly to the iso-equity curves, note that along any iso-productivity curve the value of the subsidy changes so as to satisfy the budget constraint of the government. From (13) the slope of an iso-productivity is:

$$\left.\frac{dc_H}{dc_L}\right|_{Q_S=\overline{Q}_S} = -\frac{\frac{dQ_S(c_L,c_H)}{dc_L}}{\frac{dQ_S(c_L,c_H)}{dc_H}}. \tag{14}$$

Since both c_L and c_H have a positive impact on w_S, this slope is negative. If we reduce c_H (respectively, c_L), to hold Q_S constant we have to increase c_L (respectively, c_H).

A second efficiency objective consists of raising the average level of human capital of the entire cohort of individuals, and not only that of college graduates. Recall that a proportion $1-\pi$ of the cohort has productivity $w_U(c_L)$, while those attending college have productivity $w_S(c_L, c_H, a)$. The average productivity $Q_T(c_L, c_H)$ is:

$$Q_T(c_L, c_H) = (1-\pi)w_U(c_L) + p\int_{\widehat{a}}^{A} w_S(c_L, c_H, a) f(a)da, \tag{15}$$

since only a proportion p of those with ability above the threshold $\widehat{a}$ can afford a college education. Using the definition of Q_S above, we can also write Q_T as:

$$Q_T(c_L, c_H) = (1-\pi)w_U(c_L) + \pi Q_S(c_L, c_H), \tag{16}$$

where the first term captures the aggregate level of human capital of unskilled workers and the second term takes into account both the quantity and the quality of college graduates.

The following lemma shows that the level curves of Q_T are steeper than those of Q_S in the space (c_L, c_H). Although the proof is quite simple, it is worth stating formally since it facilitates considerably the analysis of the rest of the paper.

Lemma 3 *The relationship between the slopes of the two measures of efficiency Q_S and Q_T in the space (c_L, c_H) is as follows:*

$$\left.\frac{dc_H}{dc_L}\right|_{Q_T=\overline{Q}_T} \le \left.\frac{dc_H}{dc_L}\right|_{Q_S=\overline{Q}_S} \le 0. \tag{17}$$

Proof From Eq. (16) we see that Q_T is a convex combination of $w_U(c_L)$ and $Q_S(c_L, c_H)$. In the policy space (c_L, c_H) the level curves of Q_S have negative slope, while those of $w_U(c_L)$ are vertical lines. The level curves of Q_T must have, therefore, negative slope and, at any point (c_L, c_H), they are steeper than the corresponding level curves of Q_S. □

One important implication of this lemma that we will use extensively is summarized in the proposition below. Observe from Eq. (16) that, in contrast with Q_S above, now we care also for the productivity of unskilled workers and for college attendance. In particular, consider a policy reform that raises Q_S by transferring resources from college to basic education. If college attendance π does not change, then Q_T will rise as well. However, in general we should expect a change in college attendance. If college attendance gets lower, the term on the left $((1-\pi)w_U(c_L))$ gets higher, while the term on the right (πQ_S) can either rise or fall. Nevertheless, Proposition 1 below proves that the final effect of the above reform is also positive on Q_T.

Proposition 1 *Consider any policy that raises Q_S by transferring resources from college to basic education. Then, it will also have a positive effect on Q_T.*

Proof It is immediate from Eqs. (16), (13) and Lemma 3. □

Proposition 1 tells us that, if we identify a condition under which transferring resources to basic education has a positive effect both on equity and Q_S, we know that the effect on Q_T will be positive as well. Moreover, according to Lemma 2, this is the only policy that is not discarded a priori, when looking for Pareto improvements. Thus, we can focus on Q_S, which makes our analysis much simpler. In addition, given the current trend in most Western countries towards cutting expenditure in higher education, we are interested in studying when it is the case that the policy that makes equity and efficiency compatible consists of raising c_L and reducing c_H. However, since Q_T seems to be a more standard measure of efficiency, we turn our focus to it in our numerical example of Sect. 3.5.

3.3 Equity and college productivity

The government is concerned about equity and about the quality of college graduates. We have the following definition:

Definition 1 The Equity-Efficiency Frontier (EEF) is the set of all combinations (c_L, c_H) such that, for a given threshold level of income $\widehat{y}$, college productivity Q_S is maximized.

When both Q_S and $\widehat{y}$ are differentiable and quasi-concave functions (i.e., the upper contour sets of both functions are convex sets), the EEF can be seen as the set of all combinations (c_L, c_H) where the level curves of Q_S and $\widehat{y}$ are tangent to each other.[10]

[10] As long as Q_S is quasi-concave, we do not need $\widehat{y}$ to be quasi-concave. It is enough to assume that for any combination (c_L, c_H), the upper contour set of Q_S is a proper subset of the lower contour set of $\widehat{y}$. In the Appendix we discuss the issue of quasi-concavity of Q_S. We see that Q_S will be quasi-concave under mild assumptions.

That is, the EEF is defined as the set of all combinations (c_L, c_H) such that:

$$\left.\frac{dc_H}{dc_L}\right|_{\widehat{y}=\overline{\widehat{y}}} = \left.\frac{dc_H}{dc_L}\right|_{Q_S=\overline{Q}_S}. \tag{18}$$

The EEF represents all policy combinations such that the government cannot improve equity without hurting efficiency, or the other way around. It reflects the trade-off that the government faces between equity and efficiency. Below we analyze how the size of T affects the shape of the EEF.

Next we discuss the shape of the EEF in the space (c_L, c_H). This shape will depend on the corresponding shapes of both Q_S and $\widehat{y}$. Consider first the case in which c_L and c_H are strong complements in Q_S. Just for the purpose of illustration, think of $Q_S = \min\{c_L, c_H\}$. Then, raising c_H has no effect on college productivity if, at the same time, we do not increase c_L. The slope of EEF will be positive in the space (c_L, c_H). However, for other technologies the EEF may have a different shape. For example, if money spent at early stages has a deeper impact on college productivity than expenditure at later stages, the slope of the EEF will be negative. Now putting more resources into college education has hardly any effect on productivity. If the slope of EEF is negative, a policy change that increases c_L and reduces c_H along the EEF, will improve efficiency at the price of reducing equity.

Finally, observe that there is no reason why a given economy should be actually choosing an educational policy on the curve EEF. If an economy is not at a policy combination on the EEF, there is no longer a trade-off between equity and efficiency and there is always a policy that improves the two objectives of the government. In addition, from (18) it can be checked that this policy always consists of transferring resources from college to basic education when the initial combination is to the left of the EEF, or from basic to college education if it is to the right of the EEF.

3.4 An illustration

The model so far is too general to derive policy recommendations. In the sequel, we focus on a particular example. We propose the following functional forms for $w_U(c_L)$ and $w_S(c_L, c_H, a)$:[11]

$$\begin{aligned} w_U(c_L) &= (c_L)^\alpha \\ w_S(c_L, c_H, a) &= w_U(c_L) + (c_H)^\alpha a, \end{aligned} \tag{19}$$

where $0 \leq \alpha \leq 1$ captures the fact that spending may have decreasing returns. The marginal productivity of expenditure at college is higher for high-ability individuals. Using Eq. (19) above, the productivity of college graduates, Q_S, is:

$$Q_S(c_H, c_L) = (c_L)^\alpha + (c_H)^\alpha \left(\frac{\widehat{a} + A}{2}\right). \tag{20}$$

[11] In the Appendix we discuss the robustness of our results when considering general wage functions.

Next we derive a condition that characterizes whether the economy is to the right or to the left of the EEF. By identifying this condition we are able to provide a policy recommendation for any given initial combination c_L and c_H. The next proposition shows that this condition depends crucially on the values of the elasticities of $\widehat{s}(c_L, c_H)$ with respect to c_L and c_H.

Proposition 2 *Suppose that $w_U(c_L)$ and $w_S(c_L, c_H, a)$ are as in Eq. (19). Moreover, suppose that the initial combination (c_L, c_H) is not in the EEF. Then, the particular policy reform that has a positive effect both on the productivity of college graduates Q_S and on equity depends on the size of the elasticity of $\widehat{s}(c_L, c_H)$ with respect to c_L. If this elasticity is small in absolute terms, the government should raise c_L and reduce c_H. If the elasticity is large in absolute terms, the government should raise c_H and reduce c_L.*

Proof We just have to compare the slopes of Q_S and $\widehat{y}$. The slope of $\widehat{y}$ is in Eq. (10). Regarding the slope of Q_S, we use (2), (19) and (20) to get:

$$\left.\frac{dc_H}{dc_L}\right|_{Q_S=\overline{Q}_S} = \frac{\frac{d\widehat{s}}{dc_L} - \frac{\alpha(2+\delta)(c_L)^{\alpha-1}}{c_H}}{\frac{\alpha(c_H)^{\alpha-1}A+(1-\widehat{s})}{c_H} - \frac{d\widehat{s}}{dc_H}} \tag{21}$$

There are two possibilities: either the slope in Eq. (21) is smaller than the slope in Eq. (10) or it is the other way round. In the first case, the only possibility of achieving both objectives is by increasing c_L while reducing c_H. In the second case, the way to achieve both objectives is by increasing c_H while reducing c_L. Then, we see that which case prevails depends on the value of the elasticity of $\widehat{s}(c_L, c_H)$ with respect to c_L. From Eqs. (10) and (21), we check that the first case will arise as long as:

$$\frac{\frac{d\widehat{s}}{dc_L} - \frac{\alpha(2+\delta)(c_L)^{\alpha-1}}{c_H}}{\frac{\alpha(c_H)^{\alpha-1}A+(1-\widehat{s})}{c_H} - \frac{d\widehat{s}}{dc_H}} < \frac{\frac{d\widehat{s}}{dc_L}}{\frac{(1-\widehat{s})}{c_H} - \frac{d\widehat{s}}{dc_H}}. \tag{22}$$

This can be written as:

$$(2+\delta)\,(c_L)^{\alpha-1}\,c_H\frac{d\widehat{s}}{dc_H} < (c_H)^{\alpha}\,A\frac{d\widehat{s}}{dc_L} + (2+\delta)\,(c_L)^{\alpha-1}\,(1-\widehat{s})\,. \tag{23}$$

Defining the elasticities of $\widehat{s}(c_L, c_H)$ with respect to c_L and c_H (in absolute values) as $\varepsilon^s_{c_L} = -\frac{\partial\widehat{s}}{\partial c_L}\frac{c_L}{\widehat{s}}$ and $\varepsilon^s_{c_H} = -\frac{\partial\widehat{s}}{\partial c_H}\frac{c_H}{\widehat{s}}$, respectively, the expression above can be finally simplified into:

$$\varepsilon^s_{c_L} < \left(\frac{2+\delta}{A}\right)\left(\frac{c_H}{c_L}\right)^{-\alpha}\left(\varepsilon^s_{c_H} + \frac{(1-\widehat{s})}{\widehat{s}}\right). \tag{24}$$

□

Figure 2a below represents the level curves of Q_S and $\widehat{y}$ in dashed and solid lines, respectively. They are represented as straight lines just for simplicity. Note that we

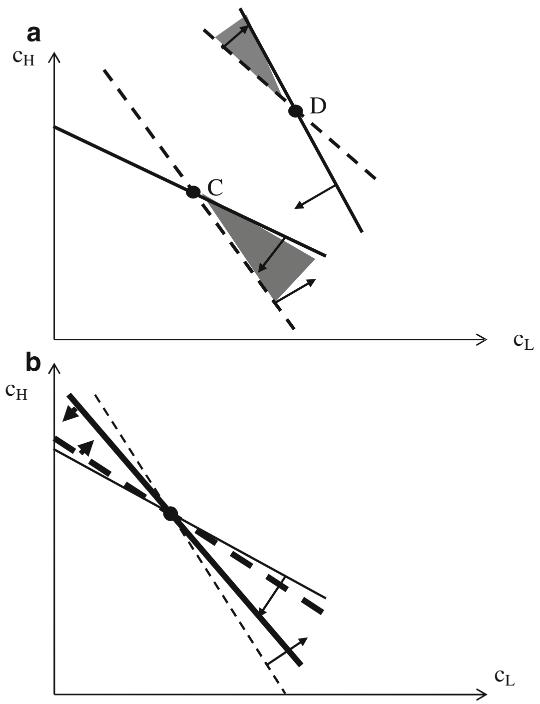


Fig. 2 **a** The optimal policy depends on the value of the subsidy. **b** The optimal policy in rich and poor countries

hold fixed parameters δ, γ, α, A, T and the income distribution. Thus, once all these parameters are fixed, each combination (c_L, c_H) is associated with a unique value of the subsidy through the budget constraint of the government. The shaded area represents the reforms satisfying both objectives. The arrows indicate policy reforms that lead to improvements in Q_S and $\widehat{y}$. Point C in Fig. 2a represents a situation in which Condition (24) holds. As we reach higher values of both c_L and c_H, the equilibrium level of the subsidy gets lower. At the same time, the iso-equity lines become steeper relatively to the iso-productivity lines. That is, as we move farther away from the origin, eventually Condition (24) will fail. This is point D in Fig. 2a.

Observe that Condition (24) is more likely to hold in rich countries (i.e., those with high T and Y). To see this, observe that the absolute size of $\varepsilon^{s}_{c_H}$ relative to that of $\varepsilon^{s}_{c_L}$, will be higher in rich countries, that is, $\varepsilon^{\widehat{s}}_{c_H}/\varepsilon^{\widehat{s}}_{c_L}$ rises with T. First note that this ratio can be written as follows, $\varepsilon^{\widehat{s}}_{c_H}/\varepsilon^{\widehat{s}}_{c_L} = \frac{\partial\widehat{s}}{\partial c_H}c_H/\frac{\partial\widehat{s}}{\partial c_L}c_L$. If the term $(\frac{\partial\widehat{s}}{\partial c_H})/(\frac{\partial\widehat{s}}{\partial c_L})$ rises with T, the ratio $\varepsilon^{\widehat{s}}_{c_H}/\varepsilon^{\widehat{s}}_{c_L}$ will rise as well. But this is precisely what happens, since iso-subsidy curves are flatter the higher is T (recall that the slope of iso-subsidy curves is $-(\frac{\partial\widehat{s}}{\partial c_L})/(\frac{\partial\widehat{s}}{\partial c_H})$). In addition, we may wonder how changes in the primitives of the model affect Condition (24). For example, we see that the higher is γ (that is, the better are capital markets), the more likely that the condition will be met. The reason is that, ceteris paribus, the higher is γ the lower is $\widehat{s}$.[12] We find a similar result looking

[12] Using Eq. (7) we see that $\partial\widehat{s}/\partial\gamma < 0$,as $\partial\pi/\partial\gamma > 0$.

at the effect of the parameter Y. A higher value of Y means a richer society. Again this makes the condition weaker.

The intuition behind Proposition 2 is the following. The policy reform consisting of raising c_L and reducing c_H has two effects. First, it raises the opportunity cost of college attendance which implies that the ability threshold will rise. This improves the productivity of skilled workers. Second, it reduces the cost of acquiring higher education. The final impact on the cost paid by students $c_H(1-s)$ depends on whether the country is rich or poor. As we have shown above, in a rich country college attendance is high, and the effect of a change in c_H relative to a change in c_L will have a deeper impact on the subsidy.[13] Thus, by reducing c_H the policy maker reduces the threshold level of income needed to attend college. Observe that this is not true for poor countries. In poor countries college attendance is low, and the effect of a change in c_L on the subsidy is stronger relative to a change in c_H. As a result, by reducing c_H and raising c_L the subsidy diminishes and this policy reform might even raise the threshold level of income required to attend college. In the next subsection we run some numerical simulations that seem to confirm this intuition.

Figure 2b above represents the policy reforms for two countries: one rich and one poor. In both cases, dashed and solid lines represents the slopes of Q_S and $\widehat{y}$, respectively. The rich country (in thin line) has an educational budget T. The poor country (in thick line) has educational budget $T' < T$. Thus, for a fixed combination (c_L, c_H), the subsidy in the rich country will be higher than in the poor country and the iso-productivity lines will be steeper than the iso-equity lines. The policy reform will consist of increasing c_L. In the poor country, since the same combination (c_L, c_H) corresponds to a much lower subsidy, the optimal policy reform is just the opposite one.

Now suppose that, instead of Q_S, we take Q_T as our efficiency measure, where Q_T is average productivity across the population. If Condition (24) holds, from Proposition 1, any policy that transfers resources from college education to basic education and that has a positive effect on equity and Q_S will have a positive effect on Q_T as well. This tells us that Condition (24) is a sufficient condition for the existence of a policy reform that moves resources from college to basic education that has a positive effect on equity and efficiency, irrespective on whether efficiency is measured by Q_S or Q_T.

Finally, what about when Condition (24) does not hold? When the slope of $\widehat{y}$ is steeper than that of Q_S there are two possibilities regarding the relationship between the slopes of $\widehat{y}$ and Q_T. The first case is when Q_T is steeper than $\widehat{y}$. This happens for middle-income countries. Then, there is always a policy reform that improves Q_T while reduces $\widehat{y}$. This policy consists again on transferring resources from college to basic education. This reform is, however, detrimental to Q_S. The second case is when $\widehat{y}$ is steeper than Q_T. This happens for low-income countries. Now, if we transfer resources from basic education to college education the three objectives are improved. As a conclusion, we find that it is always possible to find a policy reform that reduces $\widehat{y}$ and, at the same time, raises Q_T.

[13] Barro and Lee (2010), for instance, find that the fraction of population who completed tertiary education is 14.5% in advanced countries, and it is only 5.1% in developing countries.

Table 2 Parameter values (I)

δ	γ	α	A
0.1	0.75	1	1

3.5 A numerical example: high income versus low income countries

We present a numerical example to illustrate Proposition 2. We need values for δ, γ, α, A and T. Once we have this, for every combination (c_L, c_H) we can compute the values of the subsidy, college attendance and productivity. Table 2 presents our choice of parameter values. Below we describe briefly our choices.

The value $\delta = 0.1$ means that the working life of an unskilled worker is a 10% longer than that of a skilled worker. We choose $\gamma = 0.75$, since we think that borrowing constraints are not very important for most OECD countries. Regarding α, we choose $\alpha = 1$ since this simplifies very much our computations. Below we comment how results change when we allow for strictly decreasing returns, i.e., $\alpha < 1$. Regarding A, its value is related to the value of the college premium. The college premium for an individual with ability a is the ratio between net lifetime income attending and not attending college. Since a follows a Uniform distribution on $[0, A]$, the average college premium can be written:

$$\frac{2 + \delta + (c_H/c_L)(A - (1 - s))}{2(1 + \delta)}. \tag{25}$$

We use OECD data in Table 1 for average ratio $c_H/c_L = 0.61$ and subsidies for higher education, $s = 0.782$. If we fix $A = 1$, the average college premium of 1.17, which seems reasonable. Moreover, choosing $A = 1$ simplifies our calculations to a great extent.[14]

Countries are classified as rich or poor according to the values of T and Y. For each country in Table 1 we calculate T as follows. We take country data for c_L, c_H, subsidies (Column Subs.) and college attendance levels (C.Att.), and plug these numbers into Eq. (6) to get a value for T. Next, we split countries into two groups, according to their values of T. In particular, we consider as poor countries those five in the first quartile, namely: Greece, Hungary, Mexico, Slovak Republic and Turkey. We consider as rich countries the rest of the OECD countries. Observe that rich and poor countries differ not only for T, but their income distributions are also different.[15] For rich countries

[14] We also solved the numerical example with $A < 1$ for poor countries (in particular, we set $A = 0.8$) finding similar results. In particular, the lower is A, the larger is the region in the space (c_L, c_H) where Condition (24) holds. In other words, the lower the college premium (the lower the productivity increase after college attendance), the larger is the region where the optimal policy to improve both efficiency and equity consists of transferring resources from college to basic education.

[15] For simplicity we assume that y follows a uniform distribution on $[0, Y]$. Using the data in Table 1 for c_L, c_H and subsidies we compute a value of $\widehat{a}$ for each country. Using these values of $\widehat{a}$ and given college attendance levels (C.Att.) we can obtain a value of p for each country. From Eq. (3) and since $p = 1 - F(\widehat{y}) = 1 - \frac{\widehat{y}}{Y}$, once we have a value for $\widehat{y}$ and p we can also compute Y again for each particular country. Thus, the distribution of income differs across countries.

Table 3 Parameter values (II)

Country type	T	Y	c_L	c_H
Rich	100	9	70	40
Poor	40	1.5	35	25

not only T is higher than for poor countries, but also mean income Y. Table 3 shows the values of T, Y, c_L, and c_H for rich and poor countries in thousand dollars.

Once we have values for all our parameters, we can compute the equilibrium values of $\widehat{s}$, p and π for the two groups of countries. Next, we derive the shape of the EEF. To do this we start by fixing a particular value of c_L. Then, we compute the value of c_H that takes the economy to the EEF. In other words, we find the value of c_H such that the slopes of Q_S and $\widehat{y}$ coincide. Next, we repeat the process with a different value of c_L. By moving c_L through all its support, we can obtain the shape of the whole EEF.

What we obtain in our numerical example is that the slope of the EEF is negative. This result is in line with the interpretation we gave to Condition (24) after Proposition 2. In fact, we could think of the EEF curve as a way of separating those combinations (c_L, c_H) where the subsidy is too large and the condition holds (those to the left of the curve), from those where the subsidy is too low and the condition does not hold (those to the right of the curve).

As we commented in Sect. 3.4 above, changes in the primitives of the model affect Condition (24). In Fig. 3 we see that the EEF curve for rich countries (in thin line), is above the EEF curve for poor countries (in thick line).[16] Moreover, if we focus on the rich countries, the region where Condition (24) fails (those combinations above the EEF curve) corresponds to extremely low values of college attendance. This allows us to conclude that the empirically relevant region for high income countries corresponds to the situation where Condition (24) holds. However, this is not the case for poor countries, which confirms the result illustrated in Fig. 2b. As we move farther away from the origin, Condition (24) will eventually fail. The optimal reform for poor countries will consist of reducing c_L. The college subsidy will increase so that a higher proportion of poor individuals can afford college, which in turn implies an increase in college attendance. Although this reform reduces the quality of basic education and the ability threshold for college students, due to the increase in c_H, the final quality of college students increases.

Finally, we allow for decreasing returns to expenditure in education ($\alpha < 1$) and repeat our exercise for $\alpha = 0.9$. We find that, the lower is α, the larger is the region in the space (c_L, c_H) where Condition (24) holds. In particular, as we show in Fig. 3, the EEF curve for $\alpha = 0.9$ (in dotted line), is above the EEF curve for $\alpha = 1$ (in solid line). In other words, the lower the marginal return to public expenditure in education, the larger the increase in c_L in order to reach the EEF curve.

To illustrate further our results, we present in Table 4 an example of the effects of two different policy changes on the different objectives of the government. We focus

[16] For both income groups, we obtain that the larger is γ, the larger is the region in the space (c_L, c_H) where Condition (24) holds.

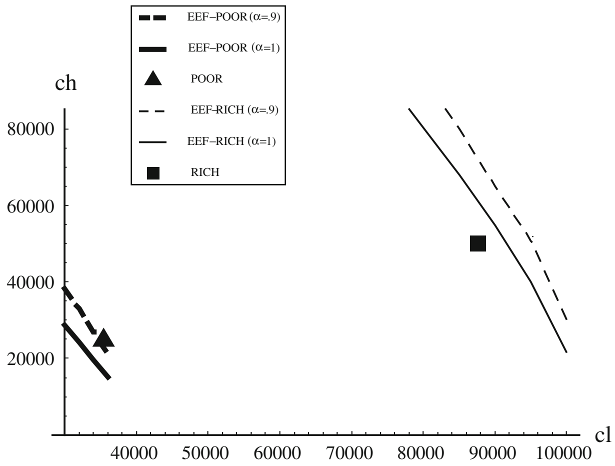


Fig. 3 Illustration of Proposition 2

Table 4 Budget division and public intervention

c_H/c_L	$.56 = \frac{50}{88}$	$.5 = \frac{45}{90}$	$.66 = \frac{57}{86}$
$\widehat{s}$	.739	.715	.748
p	.625	.629	.588
π	.352	.323	.352
Q_S	123,917	124,262	125,957
Q_T	100,650	101,215	100,060

c_L and c_H are in thousand dollars

on the case of a rich country and we use the numbers from Table 1 to set $c_L = \$88{,}000$ and $c_H = \$50{,}000$, respectively. The corresponding values of $Q_S(c_L, c_H)$, $\widehat{y}(c_L, c_H)$, $\pi(c_L, c_H)$ and Q_T are also computed, together with the values of $\widehat{s}$ and p. This initial situation is in the first column of Table 4.

We consider two alternative policies. In the first one we transfer resources from higher education to basic education while in the second one we do just the opposite. In particular, in the second column we consider a 9% reduction in c_H and a 2.8% increase in c_L. New values are $c_L = \$90{,}500$ and $c_H = \$45{,}500$. The subsidy falls a 3.7%. However, the proportion of individuals who can afford college increases from 62.5 to 62.9% meaning that this policy has a positive effect on equity. There is also a positive effect on both measures of productivity, but at the cost of a negative effect on college attendance. In particular, Q_S raises a 0.28%, while Q_T increases a 0.56%.

In the third column we consider a 2.27% reduction in c_L and a 14% increase in c_H. New values are $c_L = \$86{,}000$ and $c_H = \$57{,}000$. This policy has a positive impact on the subsidy (a 1.2% rise). Regarding the different objectives of the government, we find a negative effect on equity since the value of p falls from 62.5 to 58.8%. There is a positive effect on the productivity of college students Q_S (a 1.6% rise). However, the effect on the average level of productivity across the population, Q_T, is negative (a 0.6% reduction). As predicted by Proposition 2, moving resources towards higher education will have a negative effect either on equity or on productivity. On top of this, as seen in the reduction of Q_T, the increase in Q_S is not enough to compensate the reduction in the productivity of unskilled workers (c_L).

4 Robustness analysis

The above analysis has made a number of simplifying assumptions. In this section we want to discuss some of them in turn.

4.1 Choice of government instruments

In the real life governments have many policy instruments. We can think, for instance, of taxes, targeted subsidies and loans. In our model we do not consider taxes since we assume that the total educational budget per capita is fixed. This could be either because it comes from taxes raised on previous generations, or because it is financed through lump-sum taxes that do not distort the education decisions of the young. As our focus is not on a comparison of financing schemes but on the trade-off faced by the government between expenditure in the two education levels, we abstract from distortionary taxation. In particular, we focus on a situation in which there is no possibility of raising the budget for public education. We could think of a situation where governments have to cut spending as it is happening all across Europe these days. Given that restriction, there is still the possibility that the government wants to know if there are changes in the budget that can improve upon its different objectives, without increasing overall spending. A tax-transfer scheme would make the model more realistic. However, we are not interested here on the issue of income inequality, although we agree that this could be another potential objective of public intervention in education.

Second, with respect to targeted subsidies the simplest possibility to model them would be to fix a threshold for income such that only those below that threshold get the subsidy. Call this threshold y' and assume that $y' > \widehat{y}$. This means that, among those who can afford a college education (those with income above $\widehat{y}$), those with income within the interval $[\widehat{y}, y']$ get the subsidy while those above y' get no subsidy. If we take y' as exogenous, the analysis can be easily extended. Since now the rich are excluded from the subsidy, we can raise it for those below y'. This policy has the effect of reducing the threshold $\widehat{y}$, allowing some additional low-income individuals to attend college. However, there is also an additional unexpected effect. Since the subsidy is higher for all those below y', and it is zero for all those above y', now there is a different ability threshold for each income group. In particular, the ability threshold is lower for low-income people than for high-income people. It is not clear the final effect on average ability among college students. One problem with this policy is, therefore, that even when it may have a positive effect on equity its effect on productivity is not obvious because the ability threshold falls. Moreover, this policy would complicate the analysis regarding the relationship between the different government instruments. In particular, by increasing c_L or c_H the government has three possibilities to keep the budget balanced: (i) to keep the subsidy constant and reducing the income threshold below which individuals receive the subsidy (ii) to reduce the subsidy and maintain the income threshold constant and (iii) combine the previous two possibilities.

Finally, another possibility would be to consider public loans to students since they are a crucial instrument in many countries. However, by far the most important instrument of governments to subsidize higher education is direct subsidies. In the OECD countries they represent 80.9% of total public expenditure on tertiary education, while grants and student loans represent 10.2% and 8.9%, respectively (see Education at a Glance 2009, OECD).

4.2 Simplifying assumptions

In our model ability affects skilled wages, but not unskilled wages. This assumption can be relaxed easily. We could allow unskilled wages to depend on ability as well. From Eq. (3) it can be checked that we would get an ability threshold that separates those who attend college from those who do not. The only additional assumption we require is that the impact of ability on the unskilled wage is lower than on the skilled wage, which seems reasonable.

Another implicit assumption is that the government cannot observe individuals' ability, since subsidies cannot depend on ability. However, skilled wages depend on ability. Our defense is that it may happen that firms only learn workers' ability after some time. This is in line with the recent literature on employer learning (see, for example, Lange and Topel 2006). This assumption could reconcile observability by firms with the fact that government cannot condition subsidies on ability.

We also propose particular interpretations of efficiency and equity. Alternatively, we could propose to use some standard social welfare function. For instance, we could take an utilitarian approach by assuming that the government tries to increase average lifetime income within a cohort. The difference with our measure of average human capital Q_T is that now we are subtracting the monetary cost of higher education paid by students. Using this new definition of efficiency, we find that as long as the indirect effect that changes in both c_L and c_H have on s and π is not very large, any reform that transfers resources from college to basic education that has a positive effect on Q_S will also have a positive effect on the average lifetime income within a cohort. If, instead, the government were Rawlsian we find that the government should transfer resources from college to basic education. Since the government wants to select the policy that maximizes the utility of the worst-off individuals, we have to define first who are the worst-off. If, for example, we take as the worst-off those with innate ability and income far below the threshold levels, the result is straightforward. The government should transfer resources from college to basic education. As these individuals will not attend college, maximizing their utility will imply to maximize the unskilled wage which can only be achieved by increasing the expenditure on basic education.

4.3 Perfect capital markets

Throughout the paper we have assumed imperfect capital markets, i.e. $\gamma < 1$. If capital markets are perfect, $\gamma = 1$, equity is no longer a concern for the government since all individuals can attend college, irrespective of family income. The trade-off between equity and efficiency disappears. One interesting implication is that, in this case, the government could fix a large value of both c_L and c_H, such that $s = 0$ in order to achieve efficiency. That is, if capital markets work perfectly, college education should be privately financed.

4.4 Static versus dynamic model

Finally, it would be interesting to consider an extension to a dynamic setup. In this regard, recall that for developed/rich countries the proposed policy reform to improve both efficiency and equity consists of an increase in c_L and a reduction in c_H. Observe that this policy implies some "balance" between unskilled and skilled lifetime incomes and, thus, might not lead to a significantly less homogeneous income distribution after the first period (recall that in the simulations we use a uniform distribution). In addition, as this policy improves aggregate productivity it will lead to a higher mean income after the first period (and higher T). Therefore, the more likely is that Condition (24) in the paper will be fulfilled and thus, the same policy reform will be proposed in the next periods. For less developed/poor countries the proposed policy reform to improve both efficiency and equity consists of an increase in c_H and a reduction in c_L. This policy would lead to a less homogeneous income distribution as it improves college productivity at the price of reducing the human capital (and thus lifetime income) of those who do not attend college. In addition it is not clear whether or not this policy would improve aggregate productivity and thus mean income. Therefore it might not always be that the proposed policy reform will lead to higher income and to make Condition (24) more likely to be fulfilled. If it is so, then the proposed policy reform might change from an increase in c_H and a reduction in c_L to the reverse one.

5 Conclusions

The main finding of this paper is that, for developed countries, transferring resources from college to basic education has a positive effect on the average productivity in the population while at the same time reduces the income threshold for attending college, which makes college attendance more accessible for low-income individuals.

We believe that our results could be relevant for several recent debates in the literature on the economics of education. There is recent evidence that documents the early emergence and persistence of gaps in cognitive and non-cognitive skills (see among others, Carneiro and Heckman 2003). This issue is of special concern as, according to recent evidence, family environments have deteriorated (Heckman and Masterov 2004).[17] Studies that highlight the importance of increasing expenditure in

[17] In the US, the percentage of children born into, or living in, "nontraditional" families has increased greatly in the last 30 years (about 25% of children are now born into single parent homes now). "Non-traditional" families include not only single-parent families but also families where the parents are not married. The evidence found by Heckman and Masterov (2004) suggests that children raised in these types of families fare worse in many aspects of social and economic life.

early childhood care in achieving both equity and efficiency provide an interesting illustration since, obviously, at the current level of resources, the rise of expenditure at that level should be done at the expense of reducing expenditure in later educational levels (see Heckman 2006).

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6 Appendix

Discussion on the effect of c_L and c_H on the threshold $\widehat{a}$

Sign of $\frac{d\widehat{a}}{dc_L}$

We study the sign of $\frac{d\widehat{a}}{dc_L} = \left(\frac{\partial\widehat{a}}{\partial\widehat{s}}\frac{d\widehat{s}}{dc_L} + \frac{\partial\widehat{a}}{\partial c_L}\right)$. From Eq. (2) and Assumption 1 the first term is positive. The second term will be positive as long as:

$$\varepsilon_{c_L}^{w_U} > \frac{1}{1+\delta}\frac{w_S}{w_U}\varepsilon_{c_L}^{w_S}, \tag{26}$$

where $\varepsilon_{c_L}^{w_U} = \frac{\partial w_U}{\partial c_L}\frac{c_L}{w_U}$ and $\varepsilon_{c_L}^{w_S} = \frac{\partial w_S}{\partial c_L}\frac{c_L}{w_S}$ are the elasticities with respect to c_L of w_U and w_S, respectively. This will be the case as long as money spent on basic education affects mainly the productivity of those who do not go to college.

Sign of $\frac{d\widehat{a}}{dc_H}$

We study the sign of $\frac{d\widehat{a}}{dc_H} = \left(\frac{\partial\widehat{a}}{\partial\widehat{s}}\frac{d\widehat{s}}{dc_H} + \frac{\partial\widehat{a}}{\partial c_H}\right)$. Again, from Eq. (2) and Assumption 1 the first term is positive. Regarding the second term, we note that an increase in c_H has a positive effect both on the skilled wage w_S and on the cost of college $c_H(1-s)$. The increase in w_S reduces the threshold $\widehat{a}$ while the increase in $c_H(1-s)$ has the opposite effect. The effect on skilled wage w_s will prevail as long as:

$$\varepsilon_{c_H}^{w_S} > (1-s)\frac{c_H}{w_S}, \tag{27}$$

where $\varepsilon_{c_H}^{w_S} = \frac{\partial w_S}{\partial c_H}\frac{c_H}{w_S}$ is the elasticity of w_S with respect to c_H. Since the term c_H/w_S is of small size, (27) will hold except when s is very small. In all other cases, the condition will hold. So, in general we should expect $\frac{\partial\widehat{a}}{\partial c_H} < 0$. Thus, the total effect of c_H on $\widehat{a}$ will depend on which effect dominates. In particular, $\frac{d\widehat{a}}{dc_H} > 0$ if $\frac{\partial\widehat{a}}{\partial\widehat{s}}\frac{d\widehat{s}}{dc_H} > -\frac{\partial\widehat{a}}{\partial c_H}$.

By substituting the values of $\frac{\partial \widehat{a}}{\partial \widehat{s}}$ and $\frac{\partial \widehat{a}}{\partial c_H}$, that condition can be written as:

$$-\frac{c_H}{w_S}\widehat{s}\varepsilon^s_{c_H} + (1-\widehat{s})\frac{c_H}{w_S} > \varepsilon^{w_S}_{c_H}, \tag{28}$$

where $\varepsilon^s_{c_H} = \frac{\partial s}{\partial c_H}\frac{c_H}{s}$ is the elasticity of the subsidy with respect to c_H. Condition (28) is expected to hold if, after a change in c_H, the reaction of s is much stronger than the reaction of w_S. If, however, the opposite holds, and the effect on w_S is the strongest, then we will have $\frac{d\widehat{a}}{dc_H} < 0$.

Discussion on Assumption 1

Sign of $\frac{d\widehat{s}}{dc_L}$

We study the sign of the term in the numerator of $\frac{d\widehat{s}}{dc_L}$. We see that:

$$\text{Sign}\left[\frac{d\widehat{s}}{dc_L}\right] = \text{Sign}\left[-1 - \widehat{s}c_H\frac{\partial \pi}{\partial c_L}\right]. \tag{29}$$

This is negative if $\varepsilon^{\pi}_{c_L} > -\frac{c_L}{c_H}\frac{1}{\widehat{s}\pi}$, where $\varepsilon^{\pi}_{c_L} = \frac{\partial \pi}{\partial c_L}\frac{c_L}{\pi}$ is the elasticity of college attendance with respect to c_L. So, $\frac{d\widehat{s}}{dc_L}$ will be negative, except in those situations where the negative effect of c_L on college attendance π is very large. To illustrate further, we take average values from Table 1, where $\pi = 0.32, \widehat{s} = 0.782$, and the ratio $c_H/c_L = 0,61$. The condition becomes $\varepsilon^{\pi}_{c_L} > -6.55$. That is, the condition could only fail if the elasticity $\varepsilon^{\pi}_{c_L}$ is extremely large in absolute value.

Sign of $\frac{d\widehat{s}}{dc_H}$

Now we have that:

$$\text{Sign}\left[\frac{d\widehat{s}}{dc_H}\right] = \text{Sign}\left[-\pi - c_H\frac{\partial \pi}{\partial c_H}\right]. \tag{30}$$

The sign of $\frac{d\widehat{s}}{dc_H}$ is negative if $\frac{\partial \pi}{\partial c_H} > 0$. We focus, therefore, on the case in which $\frac{\partial \pi}{\partial c_H} < 0$. Defining the elasticity of π with respect to c_H as $\varepsilon^{\pi}_{c_H} = \frac{\partial \pi}{\partial c_H}\frac{c_H}{\pi}$, the sign of $\frac{d\widehat{s}}{dc_H}$ will be negative if $\varepsilon^{\pi}_{c_H} > -1$. That is, we require either that c_H has a positive effect on college attendance π or, if the effect is negative, the size of this effect cannot be too strong.

Slope of the iso-subsidies

Using the elasticities of π with respect to c_L and c_H as $\varepsilon^{\pi}_{c_L} = \frac{\partial \pi}{\partial c_L}\frac{c_L}{\pi}$ and $\varepsilon^{\pi}_{c_H} = \frac{\partial \pi}{\partial c_H}\frac{c_H}{\pi}$, the slope of the iso-subsidy can be rewritten as:

$$\left.\frac{dc_H}{dc_L}\right|_{s=\widehat{s}} = -\frac{1+\widehat{s}\pi(c_H/c_L)\varepsilon^{\pi}_{c_L}}{\widehat{s}\pi(1+\varepsilon^{\pi}_{c_H})}, \tag{31}$$

First, suppose that both $\varepsilon^{\pi}_{c_H}$ and $\varepsilon^{\pi}_{c_L}$ are zero. Then the slope is:

$$\left.\frac{dc_H}{dc_L}\right|_{s=\bar{\widehat{s}}} = -\frac{1}{\widehat{s}\pi}. \tag{32}$$

The lower is $\widehat{s}$ the higher is the absolute value of this expression. That is, the lower is $\widehat{s}$, the steeper are the iso-subsidies. Once we take into account the effect of both $\varepsilon^{\pi}_{c_L}$ and $\varepsilon^{\pi}_{c_H}$ the result will hold as long as they are of small size.

Quasi-concavity of Q_S and $\widehat{y}$

Consider a differentiable function $f(x) : R^2 \to R$. A sufficient condition for quasi-concavity is:

$$2f_1'(x)f_2'(x)f_{12}''(x) - (f_2'(x))^2 f_{11}''(x) - (f_1'(x))^2 f_{22}''(x) > 0. \tag{33}$$

To guarantee that Q_S is quasi-concave, it must satisfy the standard regularity conditions:

$$\frac{\partial^2 Q_S(c_L, c_H)}{\partial c_L^2} \le 0, \ \frac{\partial^2 Q_S(c_L, c_H)}{\partial c_H^2} \le 0, \ \frac{\partial^2 Q_S(c_L, c_H)}{\partial c_H \partial c_L} \ge 0. \tag{34}$$

That is, we require diminishing returns to both factors and a positive cross effect.

Discussion on the robustness of policy recommendations to general wage functional forms

Using Eq. (13), the productivity of college graduates, Q_S, is:

$$Q_S(c_L, c_H) = \frac{1}{(A-\widehat{a})} \int_{\widehat{a}}^{A} w_S(c_L, c_H, a)da, \tag{35}$$

where $\widehat{a} = \widehat{a}(c_L, c_H, \widehat{s}(c_L, c_H))$. Thus, the slope of Q_S is:

$$\left.\frac{dc_H}{dc_L}\right|_{Q_S=\overline{Q}_S} = -\frac{\frac{dQ_S}{dc_L}}{\frac{dQ_S}{dc_H}} = \frac{-\frac{d\widehat{a}}{dc_L}D_S - I_L}{\frac{d\widehat{a}}{dc_H}D_S + I_H}, \tag{36}$$

where $D_S = Q_S(c_L, c_H) - w_S(c_L, c_H, \widehat{a})$, $I_L = \int_{\widehat{a}}^{A} \frac{\partial w_S}{\partial c_L} da$ and $I_H = \int_{\widehat{a}}^{A} \frac{\partial w_S}{\partial c_H} da$. Using Eqs. (10) and (36), the slope of Q_S is lower than the slope of $\widehat{y}$ as long as:

$$\varepsilon^{s}_{c_L} < \left(\varepsilon^{s}_{c_H} + \frac{(1-\widehat{s})}{\widehat{s}}\right)\left(\frac{c_L}{c_H}\right)\left(\frac{\frac{d\widehat{a}}{dc_L}D_S + I_L}{\frac{d\widehat{a}}{dc_H}D_S + I_H}\right). \tag{37}$$

As $\left(\frac{c_L}{c_H}\right) \geq 1$, if $\left(\frac{d\widehat{a}}{dc_L} - \frac{d\widehat{a}}{dc_H}\right) D_S \geq I_H - I_L$, then the term $\left(\frac{c_L}{c_H}\right)\left(\frac{\frac{d\widehat{a}}{dc_L} D_S + I_L}{\frac{d\widehat{a}}{dc_H} D_S + I_H}\right)$ is larger than 1. In this case, and similarly to Condition (24), Condition (37) will be true as long as the elasticity of the subsidy with respect to c_L in absolute value is not much higher than the corresponding elasticity in absolute value with respect to c_H.

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